Literacy and Numeracy:

Two Solitudes No More

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Abstract

In this paper we draw attention to the impact of the new natural-language interface of *Mathematica* on the way we think about mathematics and how technology has changed how we can teach mathematics and how students are able to learn, verbalize, and use mathematical concepts and techniques. We then describe the basic elements of three online mathematics courses built with *Scientific Notebook*. The courses combine required levels of "literacy," the ability to understand and use language, with attainable levels of "numeracy," the ability to function "quantitatively" in a knowledge-based society. We refer to this link between literacy and numeracy as "e-numeracy."

Wolfram Alpha

Although still in its infancy, we can see how numerical operations and quantitative aspects of our lives are becoming accessible to a wide audience, thanks to the pioneering work of Wolfram Research in creating a natural-language interface to *Mathematica*, one of the leading computation systems, designed to make mathematical content, mathematical techniques, and mathematical ideas easy to grasp, manipulate, and use.

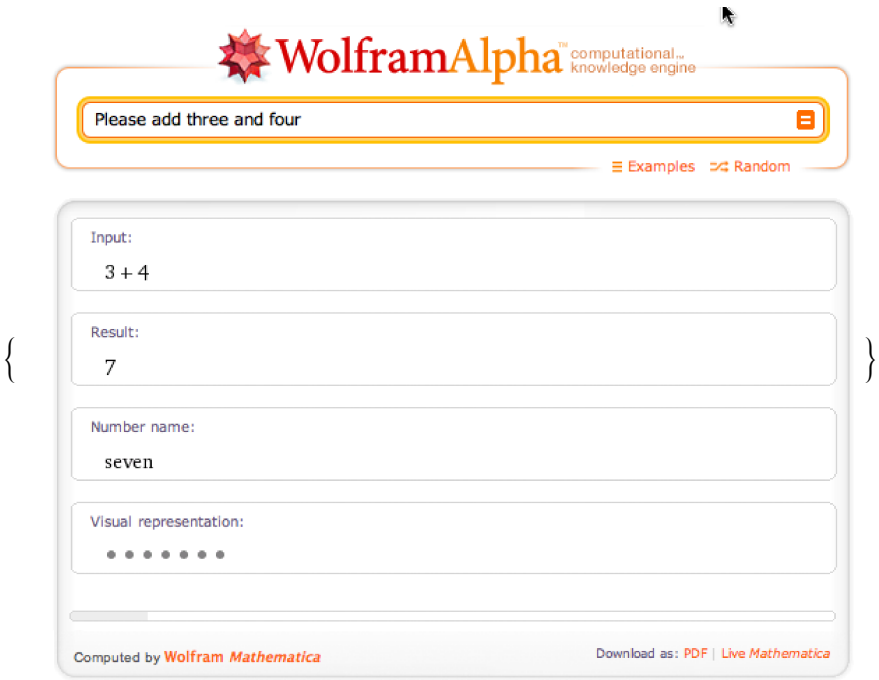


Figure 1: A Wolfram Alpha Input-Output

Acknowledgment ***The course websites discussed in this paper were produced in collaboration with KnowledgeOne, an Online Educational Services Company affiliated with Concordia University in Montreal.***

As we can see, the verbal input "Please add three and four" has produced the numerical outputs "3 + 4" and "7," as well as other relevant information such as the word "seven," the name of the output. A simple example, albeit a telling one. It relates words and numbers in a correct and meaningful way. More complicated relationships are easily produced and illustrate the power of this emerging technology.

Now let us search on “numeracy” and “literacy,” the two keywords in the title of this paper. The “numeracy” query produces a remarkable range of information:

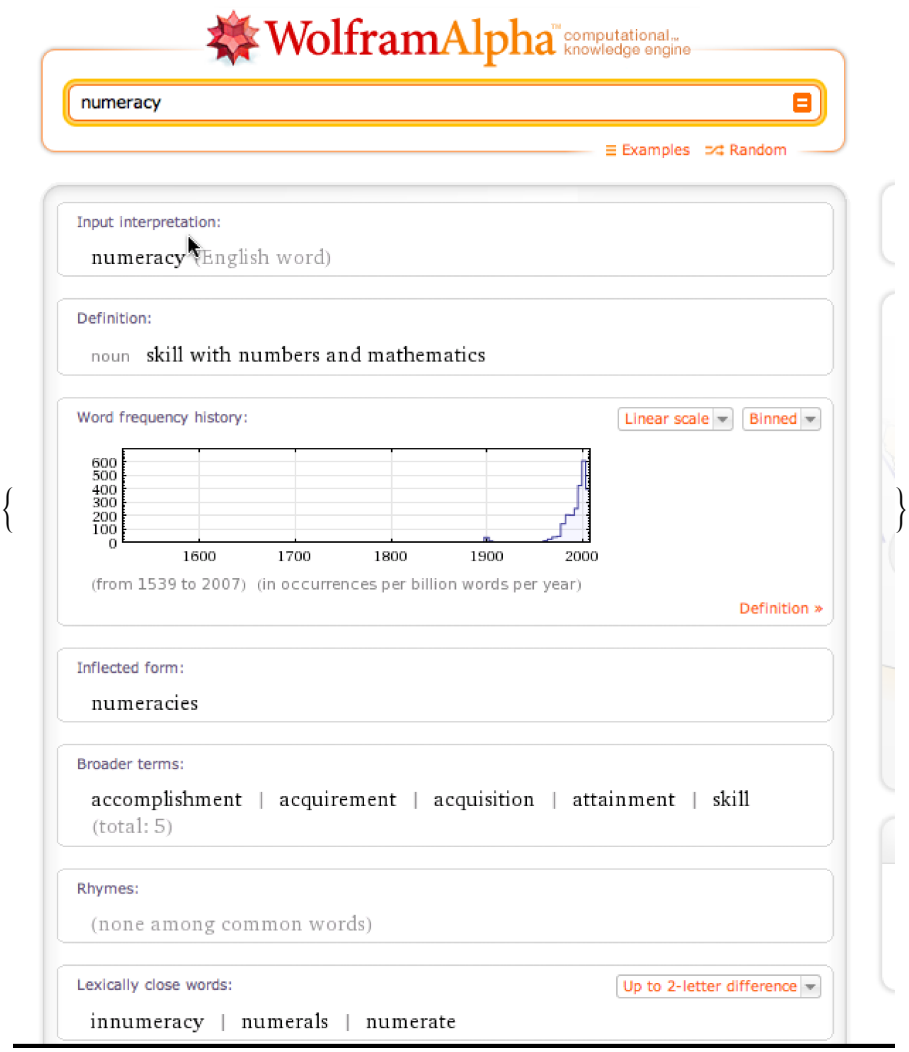


Figure 2: A Wolfram Alpha Query

First we are told how Wolfram Alpha interprets the verbal input “numeracy.” Next we are provided with a definition of the term: “skill with numbers and mathematics.” After that, a graph displays the historical frequency of the word. Wolfram Alpha also acts as a mini-thesaurus and provides a list of broader terms associated with the idea of numeracy, as well as a list of lexically close words.

The "literacy" query produces a set of complementary information. It associates the term literacy with global socioeconomic data, as well as providing numerical measures of literacy throughout the world.

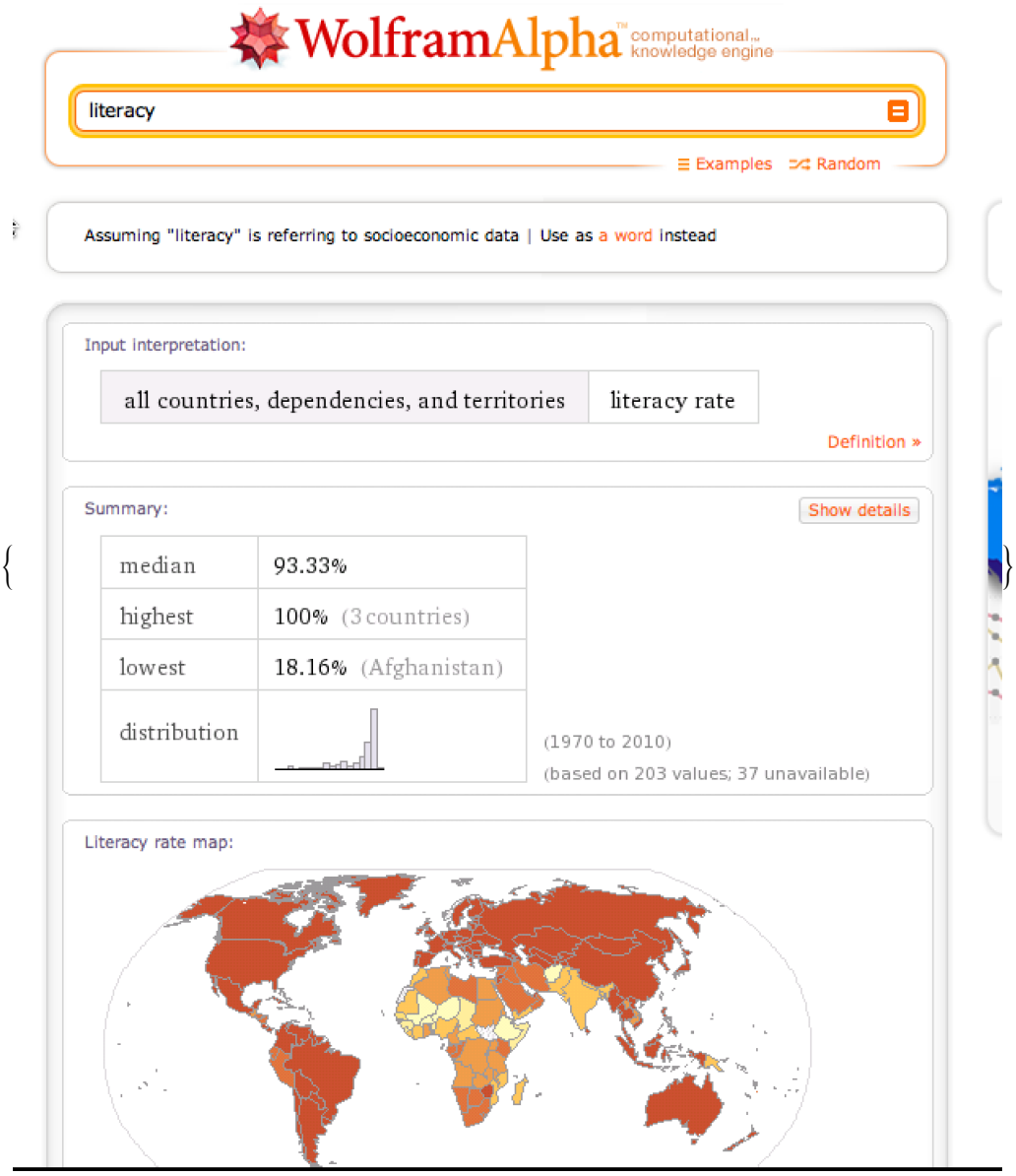


Figure 3: A Wolfram Alpha Query

Technology Impact on Teaching and Learning

Incorporating Wolfram Alpha in web-enriched classroom settings is only part of a "teaching with technology strategy." Several aspects of the predictable logic of *Mathematica* have it made relatively straightforward to recast the "do-as-I-do" blackboard-to-notebook transcription style of learning into an interactive, lucid and enjoyable experience. Here is a brief *Mathematica* dialogue:

- Compare the eigenvalues of the matrices

$$\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} \text{ and } \begin{pmatrix} 2 & 4 \\ 0 & 3 \end{pmatrix}$$

```
Eigenvalues[{{2, 0}, {0, 3}}]
Eigenvalues[{{2, 4}, {0, 3}}]
```

```
{3, 2}

{3, 2}
```

The result shows that the eigenvalues of two given matrices are identical. It illustrates the facts that the eigenvalues of a diagonal matrix are its diagonal entires and that the eigenvalues of triangular matrices are also their diagonal entries. Since the actual numerical values of the eigenvalues are irrelevant to our illustration, we can do better and use the built-in Boolean logic of *Mathematica* to produce the output we are really after:

```
Eigenvalues[{{2, 0}, {0, 3}}] == Eigenvalues[{{2, 4}, {0, 3}}]
```

```
True
```

This simple example shows how we can combine computational with conceptual goals when teaching with *Mathematica*. We can do so quickly and embed the presentation in a "what-if" setting that allows students to deepen their understanding of a topic by exploring the effect of minor (numerical) changes and adding visual components to their learning.

The conclusion of the first computation is numerical, whereas the conclusion of the second computation is conceptual, an obvious overlap of numeracy and literacy.

Let us go back to Wolfram Alpha and test whether it too can produce conceptual outputs. Instead of asking Wolfram Alpha to calculate the sum of three and four, let us ask about the equality of 3 + 4 and 7.

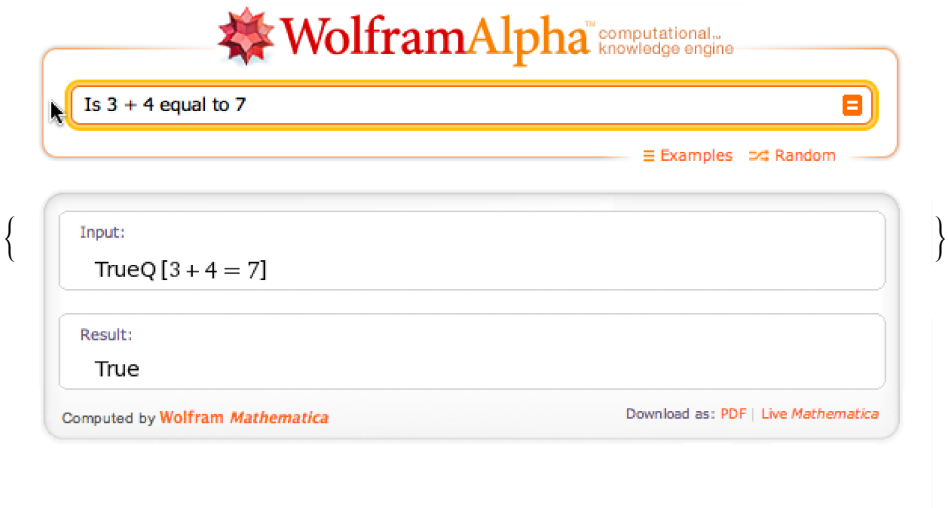


Figure 4: Wolfram Alpha Query

As we can see, the output is conceptual.

Multimedia Impact on Teaching and Learning

It is difficult to imagine how literacy and numeracy can be advanced in the modern world if we shun the use of social and multimedia to achieve these goals. Using the technological skills students have acquired through their enthusiastic embrace of slick technology provides us with an obvious starting point for other types of learning. The opportunities for developing an engaging teaching style are icing on the cake.

In this section, we illustrate the use of multimedia tools for the teaching and learning of certain types of mathematics. There is nothing special about the mathematical topics involved. Their choice was a matter of need and opportunity. However, they are the basis for a broader strategy. Three introductory undergraduate courses: a course in Linear Algebra (Math 204) based on two of my books is designed to teach linear algebra with technology: one written for Maple, and one for Mathematica. The second and third courses are the two required mathematics course for admission to the Concordia Business School: Business Mathematics (Math 208) and Business Calculus (Math 209).

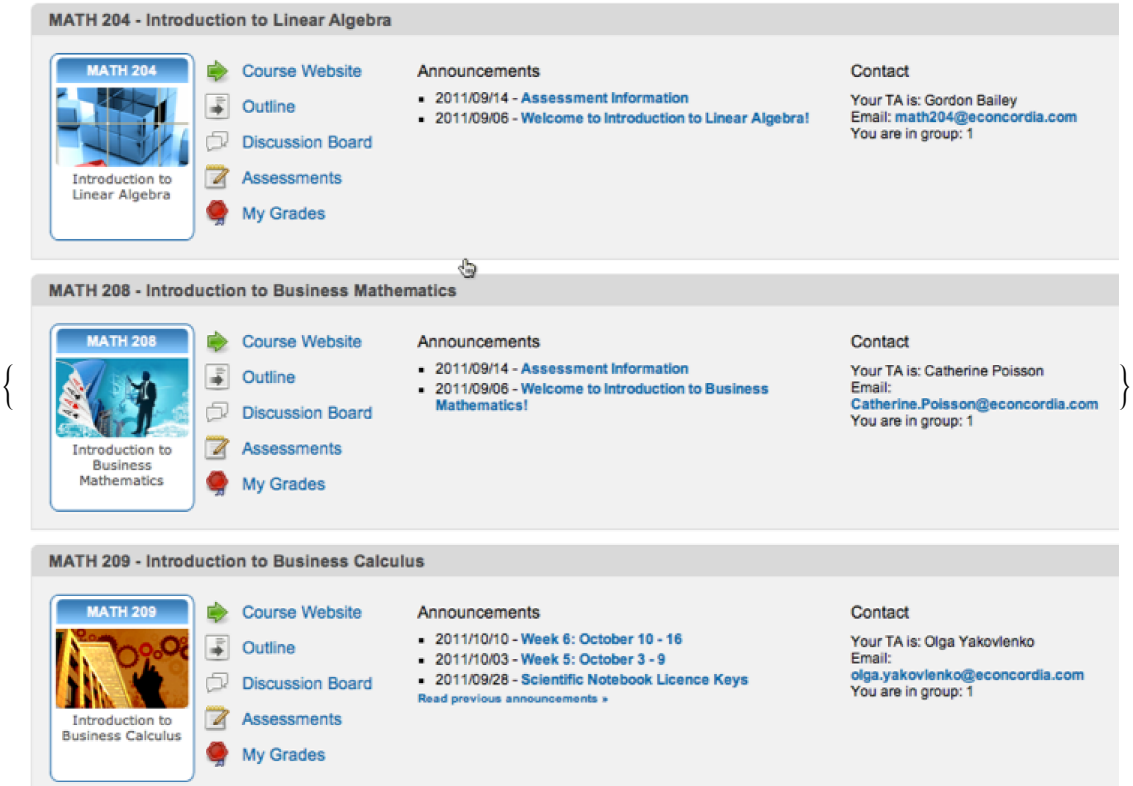


Figure 5: Online Mathematics Courses offered by eConcordia

All three courses have the same structure and the same teaching and learning elements. The following screenshots, taken from the different courses, illustrate the chosen approach.

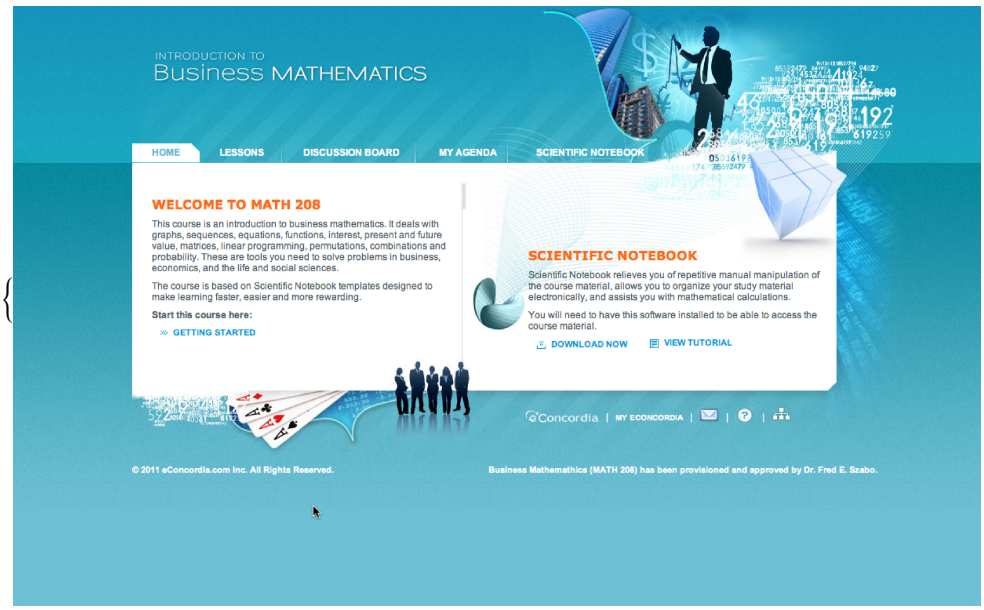


Figure 6: Home Page of Math 208

The structure of the home pages shows that they introduce the courses and provide information about *Scientific Notebook*, the software on which all three courses are based.

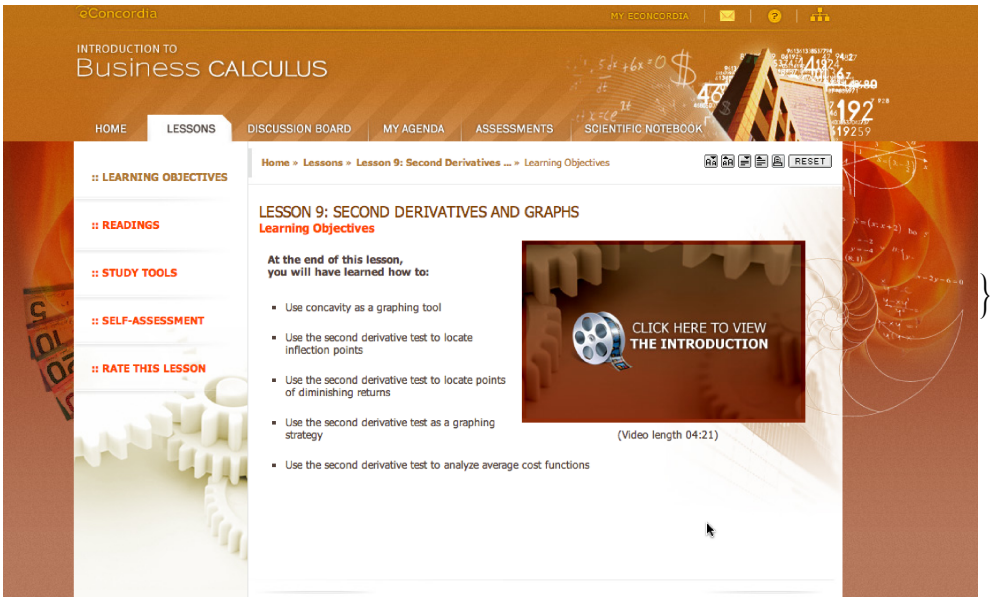


Figure 7: Lesson Page of Math 209

The learning material is organized into lessons by topics, not weeks. Math 204, for example, consists of twenty lessons, Math 208 of 12 lessons, and Math 209 of 13 lessons. In all three courses, the first lesson describes the course (About this Course) and the last lesson describes the role of the course in a more general academic setting (What Next?).

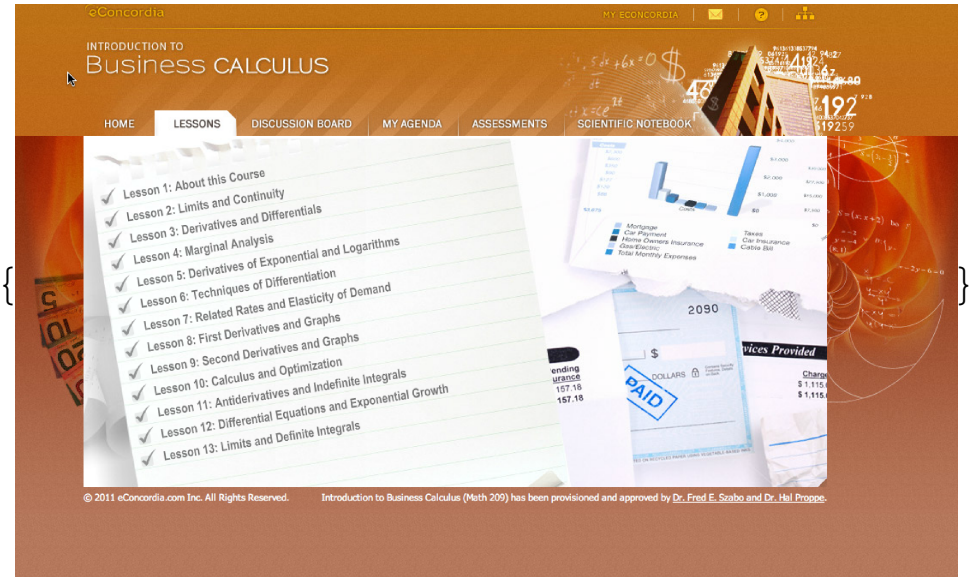


Figure 8: List of Math 209 Lessons

Each lessons consists of assigned readings, study tools, and a self-assessment section. Students are encouraged to master each lesson before proceeding with the course. All lessons are accompanies by video introductions detailing the learning objectives, highlighted with examples form the lessons. All posted material and all learning tools are created with *Scientific Notebook*. The posted material is dynamic in the sense that the mathematical elements are computable on the fly and can be experimented with and modified as the students see fit. The "what-if" aspect of learning is stressed throughout the lessons and in all courses.

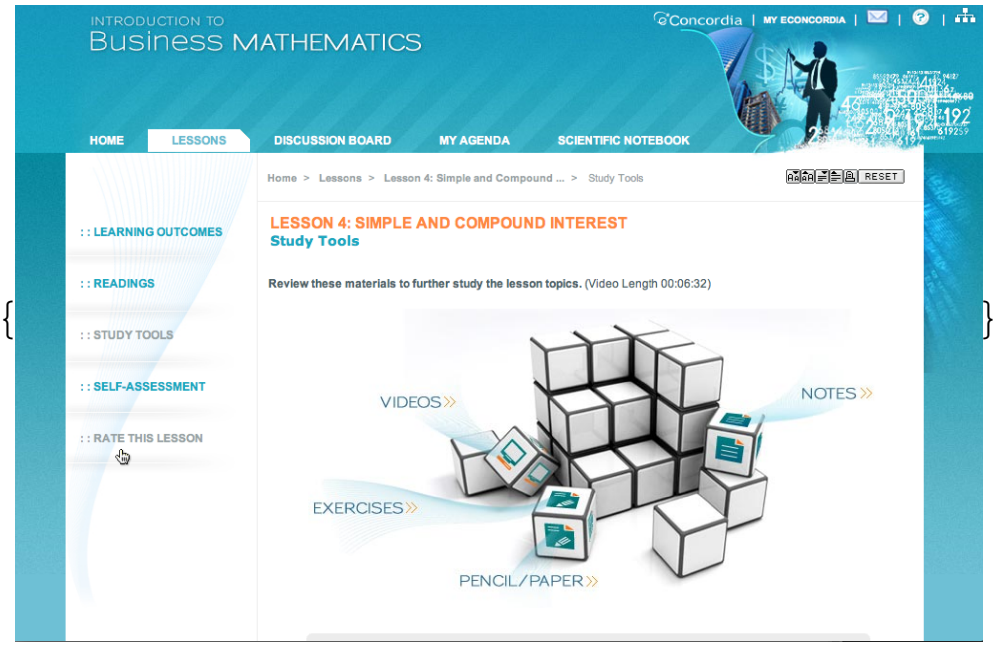
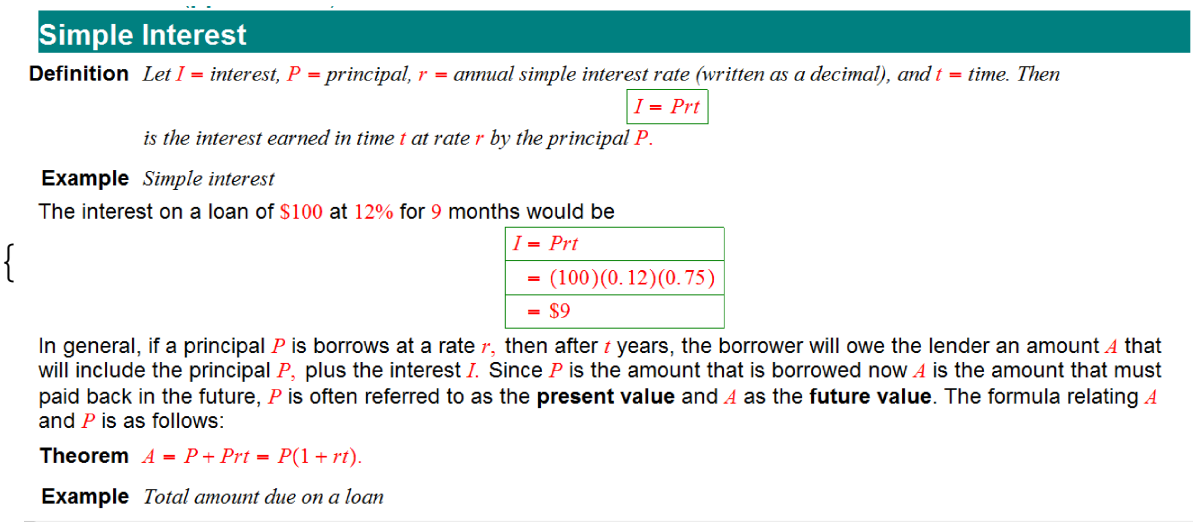


Figure 9: Component of a Math 208 Lesson

The study tools consist of dynamic lectures notes, explanatory videos, *Scientific-Notebook*-based exercises and pencil-and-paper drills to prepare the students for written final examinations in the courses. The self-assessments are randomized and designed to provide instant feedback on performance.

The following screenshot illustrates how *Scientific Notebook* is used in the three courses both to present content and to provide a platform for exploration and experimentation. The items in red are mathematically "live," and can be modified at will to produce new outputs. Changing $100 to $500, for example, and then invoking the "Compute" function, changes the $9 output to $45 in the file shown below.



Simple Interest

Definition *Let I = interest, P = principal, r = annual simple interest rate (written as a decimal), and t = time. Then*

$$\boxed{I = Prt}$$

is the interest earned in time t at rate r by the principal P.

Example *Simple interest*

The interest on a loan of $100 at 12% for 9 months would be

$$\begin{aligned} I &= Prt \\ &= (100)(0.12)(0.75) \\ &= \$9 \end{aligned}$$

In general, if a principal P is borrows at a rate r, then after t years, the borrower will owe the lender an amount A that will include the principal P, plus the interest I. Since P is the amount that is borrowed now A is the amount that must paid back in the future, P is often referred to as the **present value** and A as the **future value**. The formula relating A and P is as follows:

Theorem $A = P + Prt = P(1 + rt)$.

Example *Total amount due on a loan*

Figure 10: A Lesson Page of Math 208

All lecture notes are dynamic and self-contained. The assigned textbooks on which the notes are based act as global resources and make the courses portable to any university using the same textbooks. Since the books are also available electronically through *Course*Smart, this is obviously no limitation and Math 208 and 209 are therefore fully portable across the globe. Math 204, on the other hand, is based on the author's personal course material and the course is designed to be independent of any reference to a printed text. This course can therefore be ported lock, stock, and barel to any other college or university.

Evaluation of Students

In all three courses, the students are evaluated on two online quizzes, one online midterm examination, and one written pencil-and-paper final examination. All students take the quizzes and midterms at the same time from wherever they may be: at home, at school, on vacation, in another country across the globe, or in their departmental computing centers. The system follows a standard distance-learning model. The examinations are multiple-choice examinations and the questions are randomly chosen by the system from an expandable databank of questions, created using the graphic tools of *Scientific Notebook* and display the questions in mathematical notation.

The following screenshot illustrates the dynamic nature of the self-assessment tool in Math 209. It was externally generated from resources provided by the textbook publisher. The problems change with every attempt to take the test. The practice test therefore acts as powerful drill component of the course.

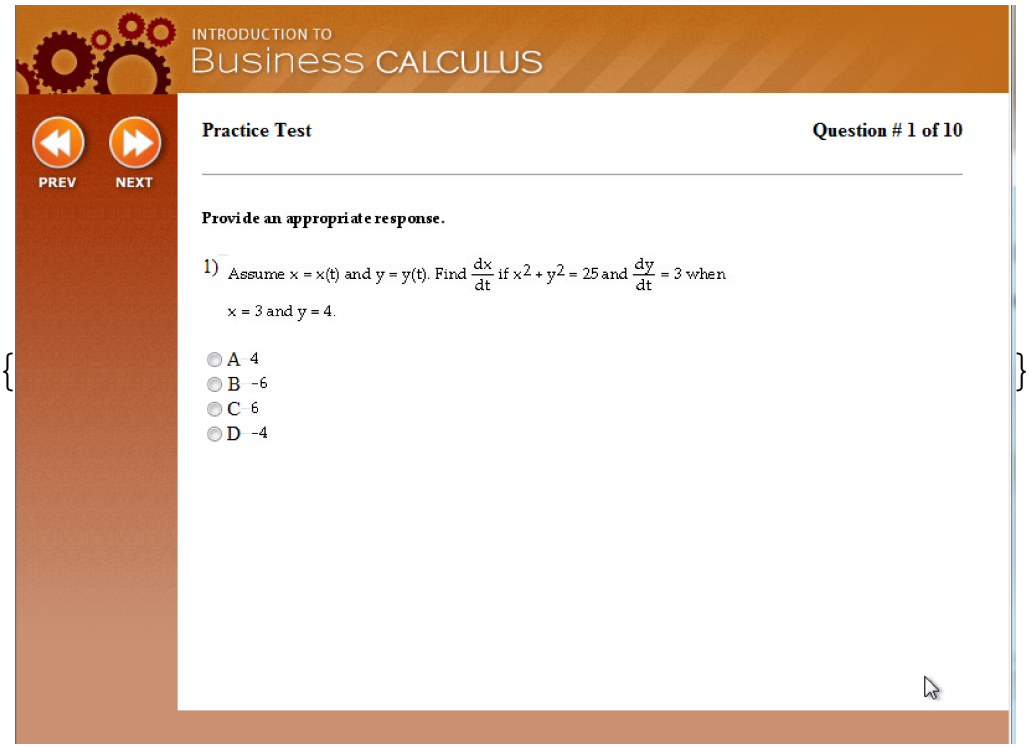


Figure 11: A Practice Test for Math 209

Cost of Production and Approximate Annual Enrolment

The first of the three discussed courses, Math 204, was first offered in January of 2006. Since then, it has been offered three times per year ever since. It took approximately 1,000 hours to develop the mathematical content, format, and examination database. This does not include the KnowledgeOne development time and cost.

The second of the three courses, Math 208, was first offered in May of 2008. Since then, it too has been offered uninterruptedly three times per year. Given the experience of developing Math 204, the development time of Math 208 was considerably shorter. It took about 600 hours to assemble, format, and structure the course material. Again, this does not include the KnowledgeOne time and cost involved.

The third of the three courses, Math 209, was first offered in September of 2009. Since then, it too has been offered uninterruptedly three times per year. It took also about 600 hours to assemble, format, and structure the course material. The development time was shared with Dr. Hal Proppe, Professor of Mathematics at Concordia University, who participated in the design and creation of this course. Again, this does not include the KnowledgeOne time and cost involved.

Course	Times Offered	Years	Total Annual Enrolment
Math 204	15	2006 – 2011	841
Math 208	11	2008 – 2011	1462
Math 209	7	2009 – 2011	723

The total annual enrolment in these courses is the number of students who completed the courses and were eligible to write the final exam. The average retention rate in these courses is about 85%. Anecdotal evidence suggests that this rate is as high or higher than the retention rate in Concordia's regular mathematics courses.

All three courses have beeen well received by the students. Most enrolled students are Concordia undergraduates, both full-

time, part-time, and independent. A small and hopefully growing number of the students enrolled in these courses come from elsewhere, both nationally and internationally.

Blending the two Solitudes

One of the pedagogical benefits of teaching and learning with technology is the growing presence and importance of language. Websites need to be searched and navigated linguistically, written instructions need to be followed and very often, as is the case in the three courses discussed in this paper, study aids such as tutorials, videos, and drills are language-based. Moreover, the entire thrust of the three mathematics courses presented in this paper is that language is not only important, it is the foundation of knowledge and learning. In this sense, online mathematics courses require a level of literacy that is probably higher than that required to transcribe cryptic mathematical information from blackboard to notebook. The fact that students can study in their own time and space also tends to deepen understanding, both linguistic and quantitative. However smart and powerful mathematical software has become, the inputting of appropriate and correct instructions requires a level of understanding that exceeds the frequently encountered jejune sketches of "solutions" to mathematical problems found in standard examination papers.

e-Numeracy

Raising the levels of "literacy," the ability to understand and use language, and of "numeracy," the ability to function "quantitatively" in a knowledge-based society, are universally heralded objectives of all modern societies. The means to achieving these goals and, in a broader sense, the feasibility of achieving them, are far from clear. However, e-numeracy is a step in the right direction. By "e-numeracy" we mean the ability to function quantitatively with the help of technology: the ability to use technology to obtain answers and solutions to mathematical problems, to understand the computed outcomes, and to be able use these results in economic, socio-political, and professional settings. How far have we come?

e-Numeracy empowers both learners and teachers: With the help of technology, the role of instructors in the teaching of mathematics has shifted from "sage on the stage" to "guide on the side." E-learning permits students to take control of their learning experience. With the help of specialized software such as *Mathematica* and *Scientific Notebook* and with appropriate web-programming, students can now interact with and manipulate the pedagogical environment at their convenience. This flexible learning strategy has revolutionized the way mathematics and other subjects can be taught and learned. In this paper, we have outlined how e-numeracy, the ability to use mathematics effectively at work, at home, and in a social setting, can be acquired by learning with technology in appropriately structured settings that enable learners to approach number-based information and formula-driven tasks with confidence, success, and enjoyment.

The eConcordia wing of KnowledgeOne, the online course provider, manages and offers over sixty online courses for Concordia University, among them Math 204, 208, and 209. In this paper we have introduced these courses to the national and internatioal academic community. KnowledgeOne is a powerful agent for advancing both literacy and numeracy in parts of the world where higher learning is inaccessible or unaffordable. The success of the three eConcordia mathematics courses shows that blending the two solitudes, intertwining the goal of raising the levels of both literacy and numeracy by teaching mathematics with technology is not only possible, it is also rewarding and has a promising future.

References

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3. Fred E Szabo, Actuaries' Survival Guide, Elsevier/Academic Press, Boston, 2004.
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7. Fred E Szabo, Actuaries' Survival Guide, Second Edition, Elsevier/Academic Press, Boston, 2012 (in press).